Mathcounts Problems !!

1. The measure of an angle is 3 times the measure of its complement. Find the measure of the angle in degrees.
2. Three calculus books weigh as much as two geometry books. Seven geometry books weigh as much as nine algebra books. Six trigonometry books weigh as much as eleven calculus books. How many algebra books weigh as much as 21 trigonometry books?
3. Notice that the product of the digits of 124 is 8. For many other three-digit positive integers is the product of the digits equal to 8?
4. Steven takes his favorite number and adds 7, multiplies the resulting number by 6, squares the resulting number, and divides by 9. The final result of these operations is 16. Given that Steven’s favorite number is not 5, what is his favorite number?
5. Given that a, b, and c are all positive, and ab = 21, ac = 18, and bc = 42, find a + b + c.
6. If x = y, find the value of 3y4 - 3(xz)2 + 3.
7. Given the sequence 2, 4, 6, 10, 16, 26..., find the 12th term.
8. Bill will be x years old in the year x. If Bill was born between 1900 and 2000, in what year was he born?
9. Five fair coins are tossed. What is the probability that exactly two heads and three tails turn up? Express your answer as a decimal.
10. Define a “strange number” to be a number where each digit (other than the leftmost two) is equal to the sum of the two digits to the left. For instance, 11235 is a strange number because 2 = 1 + 1, 3 = 1+2, and 5 = 2 + 3. How many four-digit strange numbers are there?
11. Blues, Inc. sells jeans at the following prices: $15 per pair if you buy 1 - 10 pairs, $13 per pair if you buy 11 - 40 pairs, $10 if you buy 41 - 70 pairs, and $8 per pair if you buy 71 or more pairs. For how many values of n is it cheaper to buy a number of pairs greater than n than it is to buy exactly n pairs?
12. For what value of k will the equation *x3 - 9x2 + kx* have exactly two solutions?
13. In a group of 200 students, 170 are taking history, 190 are taking math, 160 are taking English, and 135 are taking art. What is the minimum number of students that must be taking all four of these classes?
14. Find the smallest value of x that satisfies = 2.
15. At a cookie shop, four different kinds of cookies are sold: chocolate chip, macaroon, peanut butter, and white chocolate. In how many different ways can a person choose eight cookies? (Assume that the shop has an ample supply of each variety of cookie.)
16. The number can be expressed in the form - , where x and y are positive integers. Find 2x + y.
17. Bob is throwing turtles at a dartboard with two regions. If he hits the smaller region, he gets 15 points; if he hits the larger region, he gets 7 points. What is the largest score that Bob cannot get?
18. A tennis tournament starts out with 140 players. Each game is played between two players; at the end of the game, the winner advances and the loser is knocked out. There are no ties. If the tournament has a minimum number of byes, how many games must be played to determine a single winner?
19. How many positive integers less than or equal to 1000 are multiples of 2 or 3?
20. Find x2 + y2 + z2 if

*x + y + z =3*

*2x - y + z =5*

*3x +2y - z = 16*

1. Find the length of the longest median of a triangle with sides of length 4, 7, and 9.
2. If *x* + *1/x* = 8, find the value of *x4 + 1/x4*.
3. If the prime factorization of positive integer N is 24 · 39 · 11121, then how many positive integer factors does N have?
4. If 216x^2 - 4x + 7 = 16x^2 + 2x + 1, what is x?
5. For certain integers n, n2 - 3n - 126 is a perfect square. What is the sum of all distinct possible values of n?
6. The function f(x) is a cubic polynomial of the form ax3 + bx2 + cx + d. Given that f(0) = 7, f(1) = 10, f(2) = 15, and f(3) = 28, find *a +2b + 3c +4d*.
7. Suppose that f(x)=13 +23 +...+x3 and g(x) = 1+2+...+x. Compute the value of f(1)/g(1) + f(2)/g(2) +…+ f(99)/g(99).